between the data and the single straight line is encouraging, particularly in view of the different solid mixing conditions in the two runs. Some scatter in the data is to be expected, principally because of the large percentage error in the ΔTC measurement when ΔTC is small. Estimation of the film density as 1.05 g/cm^3 gives $A = 0.16$ which, with the slope of the line in Figure 2 and the measured value of n (4.55), gives $\rho = 53\,600$ mg/l. Thus, the abscissa of Figure 2 goes to approximately $\bar{x} = 1.4$. The slight curvature shown by the data from the first run at large $\bar{x}$ is as would be expected from the model (see Figure 1).

CONCLUSIONS

The growth of biomass in a liquid fluidized bed causes it to expand in a predictable way. In general, the expansion will depend on the way the biomass is distributed over the bed particles and on the solids mixing condition. However, it can be approximated over a wide range by a linear function which is independent of these factors. This function should prove useful both for inferring the quantity of biomass in a bed and for predicting the bed height.

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NOTATION

A = buoyant density of bacterial film/buoyant density of clean particle
$g(x)$ = distribution function of bacterial film
H = bed height
n = exponent in the Richardson-Zaki correlation
ΔTC = inlet total carbon concentration-outlet total carbon concentration
t = time, hr
u = particle settling velocity
v = clean particle volume
V = bed volume
v' = particle volume/liquid volume in reactor containing no bacterial film
x = film volume/clean particle volume
$\bar{x}$ = mean value of $g(x)$
y = fraction of clean particle volume below a point in the bed
ϵ = bed porosity
ρ = mg carbon/l bacterial film
τ = space time of reactor containing clean particles

Subscripts

c = bed of clean particles
o = condition at time zero
max = maximum value

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Effectiveness of Bidisperse Catalysts

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Many of the supported porous catalysts have a bidisperse pore structure. Such catalyst pellets are formed by the agglomeration of porous particles. Pores within these particles are usually called micropores, and pores between the agglomerated particles are called macropores. In such catalysts, most of the active centers lie within the particles in the micropore region. Hashimoto et al. (1976) presented a method to predict the effective diffusivities both in the macro and micropore regions of bidisperse catalysts. It has also been shown by Uyanik (1977) that both macro and micropore diffusivities can be determined by the method of single pellet chromatography which is originally developed by Doğu and Smith (1975). Diffusion and adsorption in bidisperse porous catalysts are studied by Hashimoto and Smith (1974). Wakao and Smith (1964) derived an expression for the effective diffusivity for diffusion in bidisperse porous catalyst pellets under reaction conditions. They showed that this diffusivity is a function of the effectiveness factor of the microporous particles. They also discussed the need of knowledge of the pore size distributions to predict the effectiveness factors. Silveston and Hashimoto (1971) incorporated the pore size distribution into the evaluation of effectiveness factors. Mingle and Smith (1961) derived the microeffectiveness factors for several pore distribution functions for a nonisothermal pellet. Carberry (1962) evaluated the effectiveness factor for the reversible first-order reaction. Diffusion and reaction in porous catalysts are reviewed by Aris (1975) in detail.

The effectiveness of bidisperse catalysts depend upon the rate of diffusion of reactants and products both in the macro and micropore regions as well as the rate of reaction. This implies that the prediction of the effectiveness factor of such catalysts from a single parameter, namely, the Thiele modulus (Petersen, 1965), may give erroneous results.

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Assuming that the microporous particles are spherical, and considering an n^{th} order surface reaction, we can write the volume averaged pseudo homogeneous conservation equation for these particles as

$$\frac{D_i}{r^2}\frac{d}{dr}\left(r^2\frac{dC_i}{dr}\right)-\rho_p'Sk_rC_i^n=0 \tag{1}$$

The volume averaged pseudo homogeneous conservation equation for the macroporous region can be written as (the pellet geometry is also assumed to be spherical)

$$\frac{D_a}{R^2}\frac{d}{dR}\left(R^2\frac{dC_a}{dR}\right)-\frac{3(1-\epsilon_a)}{r_o}D_i\left(\frac{dC_i}{dr}\right)_{r=r_o}=0 \tag{2}$$

Equations (1) and (2) and the suitable boundary conditions are written in dimensionless form as

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\psi_i}{d\xi}\right)-\phi^2{}_{i,n}\psi^n=0 \tag{3}$$

$$\frac{1}{\zeta^2}\frac{d}{d\zeta}\left(\zeta^2\frac{d\psi_a}{d\zeta}\right)-\alpha\left(\frac{d\psi_i}{d\xi}\right)_{\xi=1}=0 \tag{4}$$

$$\text{at}\quad \xi=0;\quad \frac{d\psi_i}{d\xi}=0 \tag{5}$$

$$\text{at}\quad \xi=1;\quad \psi_i=\psi_a \tag{6}$$

$$\text{at}\quad \zeta=0;\quad \frac{d\psi_a}{d\zeta}=0 \tag{7}$$

$$\text{at}\quad \zeta=1;\quad \psi_a=1 \tag{8}$$

where

$$\phi_{i,n}=r_o\left(\frac{\rho_p'Sk_rC_o^{n-1}}{D_i}\right)^{1/2} \tag{9}$$

$$\alpha=3(1-\epsilon_a)\frac{D_i}{D_a}\frac{R_o^2}{r_o^2} \tag{10}$$

R_o is the pellet radius and C_o is the surface concentration which may be assumed as bulk gas phase concentration if the mass transfer resistance from fluid to the external surface of the pellet is negligible. Looking at these equations, one can predict that the behavior of this system depends upon the numerical magnitudes of two parameters (Thiele modulus) $\phi_{i,n}$ and α.

If we consider a first-order reaction ($n = 1$), the solution of this system of equations is possible (Örs, 1977):

$$\psi_i=\frac{1}{\xi}\left(\frac{\psi_a}{\sinh(\phi_{i,1})}\right)\sinh(\phi_{i,1}\xi) \tag{11}$$

$$\psi_a=\frac{1}{\zeta\sinh\left[\alpha\left(\dfrac{\phi_{i,1}}{\tanh(\phi_{i,1})}-1\right)\right]^{1/2}}\,\sinh\left[\left[\alpha\left(\frac{\phi_{i,1}}{\tanh(\phi_{i,1})}-1\right)\right]^{1/2}\zeta\right] \tag{12}$$

For a first-order irreversible reaction in a single spherical porous catalyst pellet, the effectiveness factor η can be predicted from

$$\eta=\frac{9}{\phi_{i,1}{}^2\alpha}\left(\frac{d\psi_a}{d\zeta}\right)_{\zeta=1} \tag{13}$$

By substituting ψ_a from Equation (12) into Equation (13), the effectiveness factor expression for an isothermal first-order reaction taking place in a bidisperse catalyst is obtained:

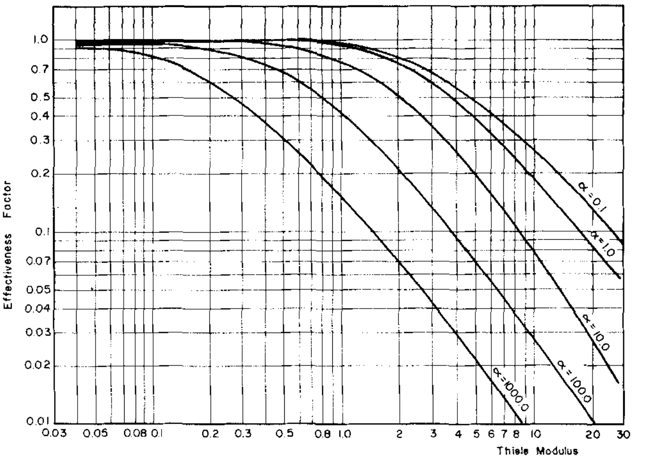

Fig. 1. Dependence of effectiveness factor on Thiele modulus and the parameter α.

TABLE 1. THE VARIATION OF THE RATIO, η/η' WITH THE PARAMETER α FOR $\phi_{i,1} = 4.0$

α	0.1	1.0	10.0	100.0	1000.0
η/η'	0.6	0.74	0.83	0.90	0.92

$$\eta=\frac{9}{\phi_{i,1}{}^2\alpha}\left\{\frac{\left[\alpha\left(\dfrac{\phi_{i,1}}{\tanh(\phi_{i,1})}-1\right)\right]^{1/2}}{\tanh\left[\alpha\left(\dfrac{\phi_{i,1}}{\tanh(\phi_{i,1})}-1\right)\right]^{1/2}}-1\right\} \tag{14}$$

The variation of effectiveness factor as predicted from Equation (14) with respect to Thiele modulus $\phi_{i,1}$ for different values of α is given in Figure 1.

It is obvious from Figure 1 that in the prediction of effectiveness factor of bidisperse catalysts, it is necessary to know the magnitude of α in addition to the Thiele modulus. The magnitude of α is actually determined by the ratio of diffusion times in the macro and micropore regions. It appears that a very wide range of values is possible for the parameter α. The magnitude of ratio of effective gas diffusivities in the micro and macropore regions is in the range of 10^{-1} to 10^{-2} for most industrially important catalysts. Of course, this ratio depends upon the pore size distributions of the pellets. Hashimoto and Smith (1974) have determined this ratio as 1.25×10^{-2} at 30°C for the diffusion of n-butane in alumina pellets with macro and micro mean pore radii of 1 200 and 17 Å, respectively. The ratio of pellet radius to particle radius lies within a wider range of values, but for most industrially important catalysts, the value of this ratio is between 10 and 100. For the pellets used by Hashimoto and Smith (1974), this ratio is about 10. If we consider these, it can be said that for most practical catalysts, the value of parameter α lies between 1 and 1 000. The value of α is about 2.5 for the pellets used by Hashimoto and Smith.

The ratios of effectiveness factor values evaluated from this model and from conventional methods are given in Table 1 for different α values having $\phi_{i,1} = 4.0$. At this value of Thiele modulus, diffusion is at least as important as reaction.

As expected, as the value of α increases, the difference between the effectiveness factors predicted from this model and the conventional model becomes smaller and smaller. At a fixed value of α, the ratio η/η' decreases as $\phi_{i,1}$ increases. For example, for $\alpha = 0.1$ and $\phi_{i,1} = 10.0$, this ratio becomes 0.32, while the same ratio is 0.6 for $\phi_{i,1} = 4.0$.

In conclusion, one can say that the prediction of the effectiveness factors from the conventional methods would give overestimated values, especially for small α.

The approach described here can also be applied to nonisothermal systems (Örs, 1977) and for other pellet geometries.

NOTATION

C_a = concentration of reactant A in the macropores
C_i = concentration of reactant A in the micropores
C_o = external surface concentration of reactant A
D_a = effective macropore diffusion coefficient
D_i = effective micropore diffusion coefficient
k_r = surface reaction rate constant
R = radial coordinate for the pellet
R_o = radius of the pellet
r = radial coordinate for the particle
r_o = radius of the particle
S = surface area per unit mass of the catalyst

Greek Letters

α = defined by Equation (9)
ϵ_a = macropore porosity
η = effectiveness factor
η' = effectiveness factor of bidisperse pellet considering it as a monodisperse one
ζ = dimensionless radial coordinate for the pellet, R/R_o
ρ_p = $\rho_p'(1 - \epsilon_a)$ = pellet density
ρ_p' = particle density
$\phi_{i,n}$ = Thiele modulus for an n^{th} order reaction
ψ_a = dimensionless concentration in the macropores, C_a/C_o
ψ_i = dimensionless concentration in the micropores, C_i/C_o
ξ = dimensionless radial coordinate for the particle, r/r_o

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Collocation Solution of Creeping Newtonian Flow Through Sinusoidal Tubes

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Laminar flow through periodically constricted tubes is usually employed as a modeling device in the study of transfer processes in porous media (Payatakes, Tien and Turian, 1973*a*, 1974*a*, *b*; Slattery, 1974; Oh and Slattery, 1976; Sheffield and Metzner, 1976; Stegermeier, 1976; Payatakes, Brown and Tien, 1977; Payatakes and Neira, 1977; Fedkiw and Newman, 1977; Payatakes, Flumerfelt, and Ng, 1977).

The problem of laminar Newtonian flow through periodically constricted tubes was solved by Payatakes, Tien and Turian (1973*b*) with a finite-difference method of the stream function-vorticity type. This method has two substantial advantages. First, it applies to tubes of arbitrary shape (allowing for wall discontinuities such as cusps, etc.) so long as the wall radius is a single-valued function of the axial coordinate. Second, it retains the non-linear inertial terms of the equation of motion, which are shown to become important at relatively low values of the Reynolds number (say, larger than 1 to 30). On the other hand, the finite difference method suffers from the disadvantages of 1) requiring large memory to achieve satisfactory accuracy, and 2) rendering the solution in matrix form, namely only on the network nodes. Stream function, velocity and pressure values at off-node points have to be calculated with two-dimensional interpolation techniques.

Hence, there are great incentives to develop analytical approximate solutions for this type of problem.

Dodson, Townsend and Walters (1971) developed

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